Chaos

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A physical system has a chaotic dynamics, according to the dictionary, if its behavior depends sensitively on its initial conditions, that is, if systems of the same type starting out with very similar sets of initial conditions can end up in states that are, in some relevant sense, very different. But when science calls a system *chaotic*, it normally implies two additional claims: that the dynamics of the system is relatively *simple*, in the sense that it can be expressed in the form of a mathematical expression having relatively few variables, and that the the geometry of the system's possible trajectories has a certain aspect, often characterized by a *strange attractor*.

Chaos theory proper, it should be noted, has its home territory in classical physics (and other kinds of dynamics that share the relevant properties of classical physics). The extent to which chaotic mathematics is fruitful in understanding the quantum realm is still a matter of debate (Belot and Earman 1997).

Sensitive Dependence on Initial Conditions In the popular imagination, a chaotic system is one whose future state may be radically altered by the smallest of perturbations—as when the fluttering of a butterfly's wings creates a disturbance whose size is inflated to the point that it tips the meteorological balance on the other side of the globe, creating a tornado where there would otherwise have been none. The "butterfly effect" marvelously engages human fear and wonder at the unpredictability of things. It captures rather less completely what is interesting and distinctive about modern chaos theory.

The idea of an inherent unpredictability in human and other affairs due to the inflation of small disturbances is an old one: "A Wise man endeavors, by considering all Circumstances, to make Conjectures, and form Conclusions: But the smallest Accident intervening, (and in the Course of Affairs it is impossible to see all) doth often produce such Turns and Changes, that at last he is just as much in doubt of Events, as the most ignorant and unexperienced Person" (Swift, *Thoughts on Various Subjects*, 1711).

Modern mathematics is able to characterize the sensitivity of initial condition dependence in various ways that lie far beyond Swift's means. Notions such as the *Liapunov exponent* help to quantify the speed at which the trajectories of systems starting out with similar initial conditions will diverge. *Measure theory* quantifies something like the chance that a small initial difference will lead to a relatively large difference in outcome, in systems where not every small change makes such a difference. There is nothing here, though, that would have astounded Swift.

Simplicity The central insight of chaos theory is that systems governed by *simple equations*, that is, systems whose behavior can be characterized by a small number of variables, or *low dimensional* systems, are often sensitive to initial conditions. At first blush this realization has a pessimistic cast. Most obviously, it leads to the conclusion that even a simple dynamics may be unpredictable in the medium to long term, as which of two significantly different outcomes occurs may depend on such fine details of the initial conditions as to lie beyond the resolving power of any reasonable observational effort.

Somewhat less obviously, certain kinds of sensitivity to initial conditions impede systematic dynamical understanding. A famous example closely connected to the origins of chaos theory is the three body problem, the task of elucidating all the properties of the dynamics of a three body system in Newtonian gravitational theory. In 1890, Henri Poincaré showed that three body systems can tend to chaos in the modern sense of the word, and concluded that a systematic treatment of three body dynamics would be very difficult if not impossible.

Chaos can be an impediment to prediction and systematic understanding in low dimensional systems, then. However, if low dimensional chaos is bad news for the study of systems known to have a low dimensional dynamics, it is good news for the study of systems known only to have a chaotic dynamics. Traditionally, such systems were modeled by complex equations, if at all; chaos theory introduces the serious possibility that these systems may be governed by equations with very few variables. Underlying complex appearances, then, may be a simple reality. The prospect of finding a hidden simplicity in such complex phenomena as turbulent flows, the weather, the movements of financial markets, and patterns of extinction is what most excites proponents of chaos theory. (Much the same prospect animates the advocates of catastrophe theory, the study of cellular automata, "complexity theory", and so on.)

To what extent can the nature of this hidden simplicity, if it exists, be divined? Given sensitive dependence on initial conditions, it is difficult to find the simple equation that best predicts the observed phenomena, since small errors in measuring initial conditions can make even the true model look like a very bad predictor. More feasible is to infer some of the more interesting properties of the putative underlying law, such as the degree of sensitivity to initial conditions and certain geometrical aspects of the dynamics induced by the law (discussed below).

In favorable conditions, this information can be used to model accurately the behavior of chaotic systems to some extent—or at least, that is the hope both of academic chaoticians and of those hoping to use the mathematics of chaos theory to beat the financial markets (as described by Bass 1999).

By far the boldest posit made in undertaking such work is the assumption that there is a simple dynamic law lying behind the subject system's complex behavior. For elaborate systems such as ecosystems and economies, the assumption of dynamic simplicity is often no more than a leap of faith; however, Strevens (2003) describes some circumstances in which ecosystems and some other complex systems have a low dimensional macrodynamics.

The Geometry of Chaos Trace the trajectory of a paradigmatically chaotic system through the space of possible states and the result is a complicated tangle of looping paths. It is the geometry of this tangle more than anything else—more even than sensitive dependence per se—that is distinctive of chaos (though there is disagreement as to which feature of the geometry is most important; see Smith (1998), chapter 10).

One especially striking feature of such trajectory tangles is their often fractal structure: they cut out a shape in the space in which they are embedded so intricate that mathematicians ascribe it a fractional dimension. Such a shape is a *strange attractor* (strictly speaking an attractor only if it is a set of trajectories that systems starting from some points outside the attractor eventually join).

Many of the more interesting properties of chaotic systems can be understood as arising from the intricate geometry of the trajectory tangle. One well-known example is the appearance of "period-doubling cascades" in systems that are moving from a periodic to a chaotic regime of behavior: as some parameter affecting the system's dynamics is tweaked, the system first oscillates between two states, then between four states, then eight states, and so on, with shorter and shorter times between each successive doubling, until it goes chaotic. What is interesting about this behavior is that it turns up in many physically quite different kinds of systems, and that there are certain aspects of the period doubling, notably the rate at which the doubling increases, that are the same (in the limit) in these otherwise rather different systems. This *universality* in chaotic systems holds out the promise of understanding the behaviors of a considerable range of systems in terms of a single mathematical—in this case, a geometrical—fact. So far, however, the wider significance of this understanding is unclear.

A more practical part of chaotic geometry is the use of limited data about the behavior of chaotic systems to reconstruct, to a certain extent, the geometry of the system's trajectory. Suppose that the behavior of a chaotic system is characterized by three variables, so that the system's "trajectory tangle" is a subset of three-dimensional space. Suppose also that only a single property of the system's dynamics can be observed, a function of the values of the three variables. In favorable conditions, this single set of observations can be used to recover the geometrical structure of the three-dimensional dynamics. Various predictions, quantitative and qualitative, can then be made from the recovered geometry.

This is an extremely powerful technique, as it assumes no knowledge of the number or even the nature of the underlying variables. However, its success does depend on, among other things, the simplicity assumption explained above: the technique supposes that there are no more than a small number of variables.

Chaos and Probability The disorderly behavior of chaotic systems can be called "random" in a loose and popular sense. Might the behavior of at least some such systems be random in a stronger sense? The suggestion that chaos might provide a foundation for probabilistic theories such as statistical mechanics has been one of the more fruitful contributions of chaos theory to philosophy.

The best scientific theories of certain deterministic, or near deterministic, systems, are probabilistic. The most prominent examples are perhaps the systems characterized by statistical mechanics and population genetics; the simplest examples are various gambling setups such as a roulette wheel or a thrown die. The probabilistic characterization of these systems is apt because the various events that make up their behavior (die throws or deaths, for example) are patterned in characteristically statistical ways, that is, in ways that are captured directly by one or other of the canonical probability distributions in statistical theory.

The mathematics of chaos offers an explanation of the probabilistic aspect of these patterns, and so offers an explanation of the success of probabilistic theories applied to certain sorts of deterministic systems.

The explanation, or rather the family of explanations, is quite complex, but it can be loosely characterized in the following way. (See Sklar 1993 for the full array of chaos-related approaches to founding statistical mechanics.) A paradigmatically probabilistic pattern has two aspects: a short term disorder, or randomness, familiar to every gambler, and a long term order that is quantified by the statistics characterizing a probability distribution, such as the one half frequency of heads in a long series of coin tosses.

Chaotic systems are capable of producing probabilistic patterns because they are capable of producing both this short term disorder and the requisite kinds of long term order. The short term disorder is due to the sensitive dependence on initial conditions; the long term order to other aspects of the "geometry of chaos", principally chaotic dynamics' resemblance to a "stretch-and-fold" process. (On the role of the stretch-and-fold geometry in producing chaotic behavior, and more specifically, probabilistic patterns, see, in increasing order of technicality, Stewart (1989), Strevens (2003), and Ornstein and Weiss (1991).)

Nowhere near all chaotic systems, it should be noted, generate probabilistic patterns. Indeed, this area of investigation is not, in a certain sense, mainstream chaos theory: there are no strange attractors or period-doubling cascades, though there is a characteristically chaotic geometry to the relevant trajectory tangles.

As well as explaining the success of probabilistic theorizing in science, chaos has been put forward—for very much the same reasons—as a foundation for the metaphysics of probability, on the principle that what produces the probabilistic pattern is deserving to a considerable extent of the name *probability*. (See Suppes (1987) and the separate entry for *probability*.)

Philosophical Significance What is the philosophical significance of chaos? With respect to general philosophy of science, opinion is divided. Some philosophers have argued that chaos theory requires the abandoning of prediction as the touchstone of successful science, a new conception of the nature of scientific explanation, and the end of reductionism (Kellert 1993). Others have argued that these conclusions are too extreme, and that insofar as they are justified, chaos theory is not necessary for their justification, though it may well have brought to philosophy's attention problems previously wrongly ignored (Smith 1998).

With respect to certain foundational questions about science, the significance of chaos is less controversial. The notion of determinism (Earman

1986), and in the context of processes that are deterministic deep down, the notions of randomness and probability, cannot be discussed without reference to work on dynamical systems since Poincaré that falls within the ambit—broadly conceived—of chaos theory.

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