3.6A Long Term Coupling

> *Approaches to explaining stochastic independence in trials that exert an ongoing causal influence on each other*
>
> *Version of 5/15/03*

Trials are coupled in the long term if they causally interact throughout their durations. In section 3.6 of *Bigger than Chaos*, I derive a sufficient condition for the independence of trials coupled in the short term: the non-deflationary microlinearity of the coupling interaction. Are there principled reasons for which trials coupled in the long term might have stochastically independent outcomes?

Observe first of all that nothing in section 3.6 absolutely requires that the coupling between trials is short term. Thus, provided that the effect of a long term coupling between trials can be represented as a non-deflationary, microlinear transformation of the trials' IC-variables, the technique developed in section 3.6 can be used to show that long-term coupled trials have stochastically independent outcomes.

The pertinent questions are, first, whether it is realistic to suppose, in many cases, that the effect of a long term coupling can be represented as a single microlinear transformation of the IC-variables, and second, in the case of a negative answer to the first question, whether there are other techniques that might be used to demonstrate the stochastic independence of long term coupled trials. In this section I discuss both questions. Neither will be fully resolved.

The Coupled and Uncoupled Evolution Functions

The composite evolution function for two colliding coins is, I show in section 3.6, rather similar to the composite evolution function for two coins that do not collide. Why? For almost all of the time they are spinning, two coins that collide have exactly the same mechanics as two coins that do not collide. It is only at the moment of collision that the mechanics differs. Thus, the effect of the collision can be represented as a perturbation of the non-colliding mechanics. This is true in general: the effect of the short term coupling between any two mechanisms can be represented as a perturbation of the uncoupled mechanics. (It is a further question whether the perturbation is microlinear; that is not my concern here.)

Is the same line of thinking reasonable when dealing with long term coupling? Certainly, it is not always reasonable. If the mechanisms are coupled sufficiently strongly—intuitively, if they are more like one mechanism than two—then the composite evolution function may bear no resemblance whatsoever to the evolution function for the uncoupled mechanisms. This observation motivates the subdivision of the category of long term couplings into *strong* and *weak* long term couplings. When a mechanism is *weakly* coupled to another throughout a trial, its mechanics are at any moment almost those it would have if it were uncoupled, save for a slight perturbation due to the action of the other mechanism. An example would be a toss of two dice that are joined by a very elastic rubber band. When a mechanism is *strongly* coupled to another throughout a trial, its mechanics are at any moment quite different from those that it would have if it were uncoupled. An example would be a toss of two dice that are joined by a rigid rod. (Clearly, the outcomes of a roll of two dice joined in this way are not independent.) The remainder of this subsection concerns weakly coupled mechanisms; strong coupling is discussed in the next subsection.

The effect of long term weak coupling between two mechanisms, I will assume, can be broken down into a series of perturbations that each mechanism causes in the time evolution of the other. The difference between short term coupling and weak long term coupling, then, is that a mechanism that is coupled in the short term experiences only one perturbation, while a mechanism that is coupled in the long term experiences a series of perturbations.

If the effect of long term coupling is sufficiently weak that it can be represented by one or a very few microlinear perturbations, then the case can be treated like a case of short term coupling. But if the series is longer, things are more complicated. Because the nature of each perturbation in the series will be affected by the mechanism's response to the last perturbation (since that response will perturb the other mechanism, the perturber), there occurs a kind of feedback not present in the case of short term coupling. The existence of feedback, the reader might well suspect, will greatly complicate the treatment of stochastic independence in weakly coupled mechanisms—if indeed there is any such independence to be had.

But things are not quite as bad as they seem. Consider the case in which each perturbation in a series induced by weak long term coupling is microlinear. (If the perturbations are not microlinear, there is presumably no prospect of treating the case according to the techniques developed in section 3.6.) Given certain assumptions that are likely to be satisfied by at least some interesting cases (see website section 3.6B), the series of perturbations can be treated, as they were in section 3.6, as all occurring at the very beginning of the trial. Then the effect of the weak coupling will be equivalent to the effect of a series of perturbations, followed by a period of evolution according to a mechanics that is just the mechanics of the uncoupled mechanisms. Thus, the outcome map for the weakly coupled mechanisms will be the outcome map for the uncoupled mechanisms transformed by the (inverse of the) microlinear perturbations.

Because a microlinear transformation preserves microconstancy, a series of microlinear transformations also preserves microconstancy. (Or: because the composition of two or more linear transformations is also linear, the composition of two or more microlinear transformations is also microlinear.) Thus in the ideal case, weak long term coupling will not, given the other assumptions made in this section, destroy stochastic independence.

How does this argument avoid having to deal with the feedback inherent in weak long term coupling? The answer is that, although the feedback determines exactly which microlinear transformation effects the next perturbation, it does not affect the fact that the next transformation will be microlinear. But since the microlinearity of the perturbations is all that is needed for independence, the particular nature and operation of the feedback is in effect completely irrelevant to the argument for independence, and so may be ignored.

Were all couplings of interest perfectly microlinear, my argument would be complete. But, of course, perfect microlinearity is a very rare thing, and the repeated application of imperfectly microlinear transformations creates the potential for small imperfections to affect the distribution of outcomes in a cumulatively large way. The same problem arises in the treatment of chains of linked trials in section 3.7, and the solution to the problem is discussed in that section of *Bigger than Chaos*.

I will sketch the solution for the benefit of the reader. There are some imperfections in microlinearity that, in the right circumstances, will undermine the stochastic independence of outcomes. But the right circumstances for undermining are not all that common. Surprisingly, the kind of dynamics that one might expect to create the greatest feedback problem—the stretch-and-fold dynamics that characterizes chaotic systems—works to smooth out the kinks introduced by imperfect microlinearity. Some kinds of weak long term coupling, then, may harness the power of microlinearity to create stochastic independence.

Let me summarize my conclusions, qualitative and tentative though they are, about weak long term coupling and stochastic independence:

1. If the coupling between two microconstant mechanisms is sufficiently weak that it can be represented by one or only a few non-deflationary microlinear perturbations of the time evolution of the uncoupled mechanisms, a case can be made that the situation resembles a short-term coupling sufficiently closely that the outcomes produced by the mechanisms are stochastically independent.

2. If the coupling between two microconstant mechanisms can be represented, as is often possible, by a longer series non-deflationary of microlinear perturbations, all occurring at the beginning of the experiment (see website section 3.6B), the outcomes of trials on the mechanisms may be independent. The closer to perfect microlinearity the perturbations are, the more likely are the outcomes to be independent, but considerable imperfection can be rectified if the dynamics of the experiment has a stretch-and-fold aspect—if it is not merely non-deflationary, but positively inflationary.

3. If the form of the perturbations is not microlinear, or if the effect of the coupling is too great to be represented as a perturbation of the uncoupled time evolution of the mechanisms concerned, the results developed in section 3.6 are of no help in demonstrating independence.

Independence without Perturbation

I will next discuss, very informally, a set of circumstances in which even very strongly coupled mechanisms can produce stochastically independent outcomes.

The prerequisites for this result are that the mechanisms are microconstant, and that the evolution functions for each coupled mechanism have the same strike ratios as the evolution functions for the corresponding uncoupled mechanism. For example, if the mechanisms are two dice, the strike ratio for, say, a '6', on each coupled die must be the same as the strike ratio for an uncoupled die, that is, 1/6. This is a strong condition: because the evolution function for a coupled die takes into account all the physics of the strong coupling, it may have a quite different aspect from the evolution function for the uncoupled die. But without this condition, I do not think there is much prospect for any kind of independence result.

Consider two microconstant mechanisms that satisfy the condition, say, two wheels of fortune coupled by some elaborate device. The evolution function for each wheel of fortune will take as its IC-variables both its own initial spin speed and the initial spin speed of the other wheel. Any trial on the composite experiment will have the same two IC-variables.

Because I am imagining that the coupling between the wheels is very elaborate, the outcome map for each wheel can have just about any pattern you like, subject to the requirement that the strike ratio for any outcome on a wheel is equal to the strike ratio which that outcome would have if the coupling did not exist. I will take red as my designated outcome, and assume that the strike ratio for red on both wheels is 1/2. Suppose, for example, that the evolution functions for the two wheels are as shown in figure 3.6A.1. (Note that the coupling has a rather asymmetric effect! If

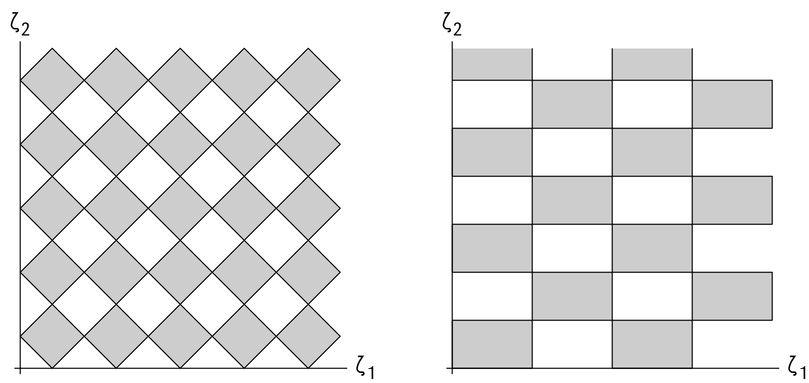

Figure 3.6A.1: The evolution functions for two strongly coupled wheels of fortune

the effect were symmetric, and the wheels otherwise identical, the outcome map for one wheel would be the map for the other reflected through the line $\zeta_1 = \zeta_2$.)

Because the uncoupled wheels are microconstant and each has a strike ratio for red of 1/2, the probability of red on either wheel when uncoupled is 1/2, assuming macroperiodicity of the individual IC-densities. Trials on two coupled wheels will be independent, then, if the strike ratio of the composite evolution function for any of the four possible pairings of colors (for example, red-red) is 1/4, now assuming macroperiodicity of the joint IC-density as well.

The composite evolution function can be constructed by superimposing the individual evolution functions and marking the parts of the space in which two shaded regions coincide. The result is shown in figure 3.6A.2. It will be seen that the strike ratio for red-red is indeed very close to 1/4.

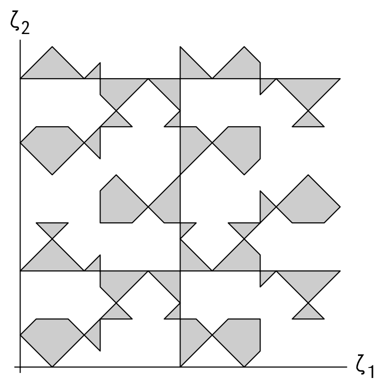

Figure 3.6A.2: The composite evolution function for the two coupled wheels of fortune

This is true for the other color pairings as well, so that the outcomes of trials on the two wheels are more or less stochastically independent.

Is there any reason for this, or is it just an accident? There is a reason of sorts. Because the repeated pattern that makes up one outcome map is quite different from the repeated pattern that makes up the other, and in particular because the size and shape of the members of any optimal constant ratio partition for one map are quite different from the size and shape of the members of any optimal constant ratio partition for the other, the regions where the two maps coincide are, as it were, a representative selection of the regions from one and the regions from the other. As a result the areas of coincidence occupy a fraction of the total area approximately equal to the product of the areas occupied by the gray areas of each map individually.

The conditions under which this will tend to be the case can be spelled out formally, but I will not attempt to do so here. I simply note that this phenomenon need not be a particularly rare one. All that is needed, aside from the admittedly strong constraint on the strike ratios, is a lack of correlation between optimal constant ratio partitions for each coupled mechanism. And a lack of correlation, though it is not necessarily to be expected, is not necessarily a surprise.