3.7B The Evolution Function and the IC-Evolution Function

The sufficient conditions for the stochastic independence of chained trials require certain properties of both the evolution function and the IC*-evolution function. The nature of both functions is determined by the same physics, and it turns out that physical properties conducive to satisfying the conditions on* IC*-evolution are also conducive to satisfying the conditions on the evolution function (principally, microconstancy).*

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The explanation of complex systems' simplicity in chapter four of *Bigger than Chaos* takes as the outcomes of microdynamic experiments sets of initial conditions for the immediately succeeding microdynamic experiments. (Recall that, although the outcome of a microdynamic experiment is an event corresponding to more than one IC-value for the next *microdynamic* experiment, it determines a unique next microstate and so corresponds to a single IC-value for an *enion probabilistic* experiment.)

Put in terms of the autophagous wheels of section 3.7 (all references are to *Bigger than Chaos*), this corresponds to taking as the designated outcome of a spin on an autophagous wheel a range of effective IC-values for the next spin (but not a single effective IC-value).[1] Thus, the outcome of the i^{th} trial is a range of effective IC-values, corresponding to an interval of full IC-values, for the $(i+1)^{\text{th}}$ trial. The aim of this section is to remark on an important consequence of taking as the outcome of one trial in a chain a range of IC-values for the next.

If the designated outcome for a chained trial is a range of effective IC-values, then the evolution function for the outcome is determined by the way that the mechanism maps its own IC-values onto the IC-values for the

1. Why not consider a single effective IC-value as the outcome? If the designated outcome of the i^{th} trial is an effective IC-value for the $(i+1)^{\text{th}}$ trial, then the effective IC-value for the i^{th} trial must determine, by definition, the outcomes of all future trials in the chain. The reason is as follows: the effective IC-value for the i^{th} trial determines the outcome of the i^{th} trial, which is the effective IC-value for the $(i+1)^{\text{th}}$ trial, which determines the outcome of the $(i+1)^{\text{th}}$ trial, which is the effective IC-value for and thus determines the outcome of the $(i+2)^{\text{th}}$ trial, and so on. Thus, assuming a fixed amount of inflation in each trial, if there is such a thing as this effective IC-value, it can only be the full IC-value.

next trial. This mapping is, of course, none other than the IC-evolution function for the trial. The IC-evolution function, then, determines the evolution function.

To obtain the independence result stated in section 3.76, I make one strong assumption about the evolution function for the designated outcome, that it is microconstant, and two assumptions about the IC-evolution function, that it is sufficiently inflationary and sufficiently microlinear. These are constraints on what is, at bottom, a single process. How do they sit together? Are they harder to satisfy jointly than individually, no harder, or is there some synergy?

There is considerable synergy: provided that the designated outcomes are chosen appropriately, if the IC-evolution function is inflationary and microlinear in the right way, then the corresponding evolution function is sure to be microconstant. Thus the conditions for independence are easier to satisfy than it might seem.

The following kinds of inflation and microlinearity in IC-evolution will entail microconstancy:

1. The inflation induced by IC-evolution should be *folding*, in the following sense (slightly stronger than the sense defined in section 3.7A): there must be a partition of the IC-values for the experiment into many sets such that each set in the partition is mapped onto the same set of full IC-values. I call the partition the *folding partition* (equivalent to the buffer partition of section 3.7A).

2. The microlinearity should be sufficiently strong that the IC-evolution function is linear over each member of the folding partition. This is considerably more microlinearity, I note, than is required of IC-evolution in section 3.76 for the independence of chained microconstant trials.

The designated outcomes should then be chosen as follows: a designated outcome for the i^{th} trial in a chain should be the event of the IC-value for the $(i+1)^{\text{th}}$ trial in the chain falling into some particular member of the folding partition (see figure 3.7B.1).

The folding partition is, I claim, a constant ratio partition for any such designated outcome. The reason is the linearity of the IC-evolution function over each member of the folding partition, which ensures that the proportion of IC-values for the i^{th} trial within any member of the

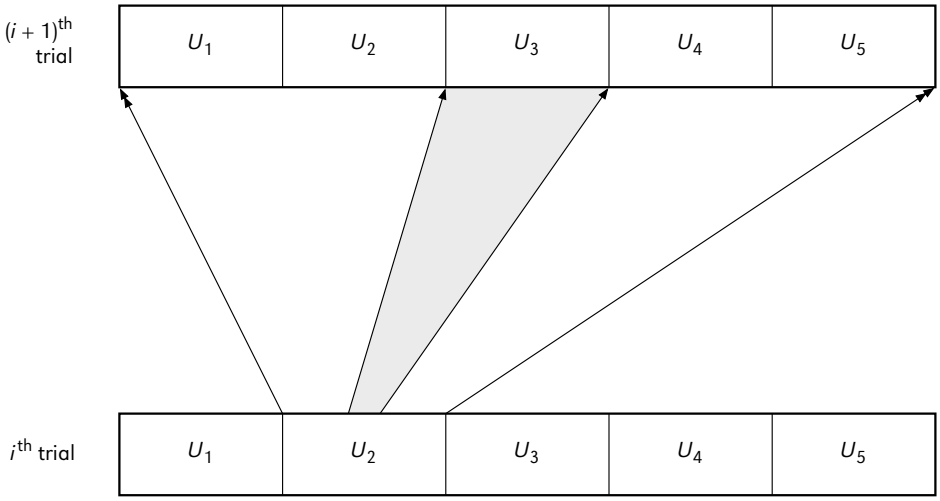

Figure 3.7B.1: The outcome of the i^{th} trial is a set of effective IC-values for the $(i+1)^{\text{th}}$ trial. The set should be a member of the folding partition for the experiment. In the figure, the folding partition is the set $\{ U_1, \ldots, U_5 \}$, meaning that each U_i is mapped by the experiment's IC-evolution function to the set of all possible IC-values (as shown for U_2 by the double-headed arrows). An outcome of the i^{th} trial, then, might be the event of the IC-value for the next trial falling into U_3. Because IC-evolution is by assumption approximately linear over any member of the folding partition, the folding partition is a constant ratio partition for such an outcome: within any partition member, the strike ratio for a given outcome U_i is the same, equal to the proportion of all possible IC-values falling into U_i.

folding partition mapping to a given set of IC-values for the $(i+1)^{\text{th}}$ trial, is the same. (It is equal to the size of the folding partition member as a proportion of all possible IC-values, as shown in figure 3.7B.1.)

Provided that the folding partition is micro-sized, then, which does not quite follow from the conditions stated above, but follows from the assumption that IC-evolution is inflationary, the folding partition is a micro-sized constant ratio partition for any designated outcome, and so the probability distribution over the designated outcomes is microconstant.

It is this overlap of the conditions for microconstancy and the conditions on IC-evolution sufficient for independence that explains the double

duty done by various derivations in the treatment of statistical physics in section 4.8 (see the comments at the end of section 4.83).

I will make one remark about the sufficient conditions for the independence of chained non-microconstant trials discussed in section 3.7A. If the IC-evolution function is microlinear in a strong enough way, then the folding requirement for independence entails the well-temperedness requirement. Microlinearity must be strong in the following sense: it is not enough that the IC-evolution function for the i^{th} trial be microlinear over the inverse image of the buffer partition for the $(i+1)^{\text{th}}$ trial; it should be microlinear over the buffer partition for the i^{th} trial. At the end of section 3.7A, I discuss ways in which the independence conditions stated in that section might be satisfied without such strong microlinearity.